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Hard Exclusive Production of Tensor Mesons

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Abstract:

We point out that hard exclusive production of tensor mesons $f_2(1270)$ with helicity $\lambda = \pm 2$ is dominated by the gluon component in the meson wave function and can be used to determine gluon admixture in tensor mesons in a theoretically clean manner. We present a detailed analysis of the tensor meson distribution amplitudes and calculate the transition form factor $\gamma + \gamma^* \to f_2(1270)$ for one real and one virtual photon.

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1. Gluon admixture to meson wave functions is interesting for many reasons and has been subject to extensive and somewhat controversial discussion over many years, see e.g. [1]. The purpose of this letter is to point out that separation of quark and gluon contributions in hard processes that are dominated by hadron wave functions at small transverse separations has a different meaning compared to the quark model; in a suitably chosen reaction quark contribution can be down compared to the gluon contribution by a power of the momentum transfer.

In particular, we consider hard exclusive production of the tensor meson $f_2(1270)$ with the quantum numbers $J^{PC} = 2^{++}$, $I^G = 0^+$. This state is non-exotic, and, in quark model, can be constructed either from a quark and an antiquark, or from a pair of gluons. To get the spin projection $s = 2$ the quark and the antiquark have to be in a P-wave state, while the gluons can be in S-wave. On the light-cone, however, contribution of the orbital angular momentum is higher-twist and we will find that the form factor $\gamma + \gamma^* \to f_2(1270)$ corresponding to a pure helicity state $\lambda = \pm 2$ is dominated at large photon virtualities by the gluon contribution.

2. Distribution amplitudes of a tensor meson can be constructed in full analogy with those of vector and pseudoscalar mesons, see e.g. [2, 3, 4, 5]. We consider matrix elements of bilocal quark-antiquark operators at a light-like separation z_μ, $z^2 = 0$:

$$\langle f_2(P,\lambda)|\bar\psi(z)\gamma_\mu\psi(-z)|0\rangle = f_q m^2 p_\mu \frac{e^{(\lambda)}_{\bullet\bullet}}{p_\bullet^2}\int_{-1}^{1} dt\, e^{itp_\bullet}\phi_q(t)$$

$$+\, f_q m^2\left[e^{(\lambda)}_{\mu\bullet} - p_\mu\frac{e^{\bullet\bullet}}{p_\bullet}\right]\frac{1}{p_\bullet}\int_{-1}^{1} dt\, e^{itp_\bullet} g_v(t) + \mathcal{O}(z_\mu)\,,$$

$$\langle f_2(P,\lambda)|\bar\psi(z)\gamma_\mu\gamma_5\psi(-z)|0\rangle = -if_q m^2\varepsilon^{\mu\nu\alpha\beta}\frac{z^\nu p^\alpha}{p_\bullet}\frac{e^{(\lambda)}_{\beta\bullet}}{p_\bullet}\int_{-1}^{1} dt\, e^{itp_\bullet} g_a(t)\,, \qquad (1)$$

where $\bar\psi\psi = \bar u u + \bar d d$. P_μ and $m = 1270$ MeV are the f_2-meson momentum and mass, respectively: $P^2 = m^2$. Here and below we use a shorthand notation $p_\bullet = p_\mu z^\mu$, etc., and define the light-like vector $p_\mu = P_\mu - z_\mu m^2/(2p_\bullet)$, $p^2 = 0$. The polarization tensor $e^{(\lambda)}_{\alpha\beta}$ is symmetric and traceless, and satisfies the condition $e^{(\lambda)}_{\alpha\beta}P^\beta = 0$. Polarization sums can be calculated using

$$\sum_\lambda e^{(\lambda)}_{\mu\nu}\left(e^{(\lambda)}_{\rho\sigma}\right)^* = \frac{1}{2}M_{\mu\rho}M_{\nu\sigma} + \frac{1}{2}M_{\mu\sigma}M_{\nu\rho} - \frac{1}{3}M_{\mu\nu}M_{\rho\sigma}\,, \qquad (2)$$

where $M_{\mu\nu} = g_{\mu\nu} - P_\mu P_\nu/m^2$ and the normalization is such that $e^{(\lambda)}_{\mu\nu}(e^{(\lambda')}_{\mu\nu})^* = \delta_{\lambda\lambda'}$.

The distribution amplitude $\phi_q(t) = -\phi_q(-t)$ is the leading twist-2 distribution amplitude for the tensor mesons with helicity $\lambda = 0$, while $g_v(t) = -g_v(-t)$ and $g_a(t) = g_a(-t)$ correspond to the twist-three amplitudes with helicity $\lambda = 1$. The leading twist-2 distribution amplitude for $\lambda = 1$ is given by a similar matrix element with the $\sigma_{\mu\bullet}$ matrix in between the quarks; it does not contribute to two-photon processes (if quark masses are neglected) and will not be considered here.

Normalization of the distribution amplitudes is chosen to be:

$$\int_{-1}^{1} dt\, t\, \phi_q(t) = \int_{-1}^{1} dt\, t\, g_v(t) = 1\,, \qquad \int_{-1}^{1} dt\, g_a(t) = 0\,. \tag{3}$$

The constant f_q is defined as the matrix element of the local operator

$$\langle f_2(P,\lambda)|\bar{\psi}\gamma_\mu i \overleftrightarrow{D}_\nu \psi|0\rangle = f_q m^2 e^{(\lambda)}_{\mu\nu} \tag{4}$$

and was estimated using QCD sum rules [6]:

$$f_q(\mu = 1\ \text{GeV}) \simeq 56\ \text{MeV}\,. \tag{5}$$

The twist-three distribution amplitudes $g_v(t)$ and $g_a(t)$ receive a Wandzura-Wilczek type contribution that can be expressed in terms of the distribution function ϕ_q of the leading twist. A standard calculation (cf. [4]) yields

$$\begin{aligned} g_v^{WW}(t) &= \int_{-1}^{t} dw\, \frac{\phi_q(w)}{1-w} + \int_{t}^{1} dw\, \frac{\phi_q(w)}{1+w}\,, \\ g_a^{WW}(t) &= \int_{-1}^{t} dw\, \frac{\phi_q(w)}{1-w} - \int_{t}^{1} dw\, \frac{\phi_q(w)}{1+w}\,. \end{aligned} \tag{6}$$

Because of a large mass of the f_2-meson, we expect that the twist-3 distribution amplitudes are dominated by these "kinematic" contributions whereas genuine twist-3 contributions of quark-gluon operators are relatively small.

The conformal expansion of the leading-twist distribution amplitude $\phi_q(t)$ goes over the usual set of Gegenbauer polynomials $(1-t^2)\, C_n^{3/2}(t)$ with odd values of $n = 1, 3, \ldots$ because of the C-parity. The asymptotic wave function is, therefore

$$\phi_q^{\text{as}}(t) = \frac{15}{4} t(1-t^2) \tag{7}$$

and the corresponding expressions for the twist-3 distributions are

$$g_v^{\text{as}}(t) = \frac{5}{2} t^3\,, \qquad g_a^{\text{as}}(t) = \frac{5}{4}(3t^2 - 1)\,. \tag{8}$$

In addition, there exist two different leading twist gluon distribution amplitudes

$$\begin{aligned} \langle f_2(P,\lambda)|S_{\mu\nu} G^a_{\bullet\mu}(z) G^a_{\bullet\nu}(-z)|0\rangle &= f_g^T e^{(\lambda)}_{\mu\nu} p_\bullet^2 \int_{-1}^{1} dt\, e^{itp\bullet} \phi_g^T(t) + \mathcal{O}(z_\mu, z_\nu)\,, \\ \langle f_2(P,\lambda)|S_{\mu\nu} G^a_{\xi\mu}(z) G^a_{\xi\nu}(-z)|0\rangle &= f_g^S e^{(\lambda)}_{\mu\nu} \int_{-1}^{1} dt\, e^{itp\bullet} \phi_g^S(t) + \mathcal{O}(z_\mu, z_\nu)\,. \end{aligned} \tag{9}$$

Here $S_{\mu\nu}$ stands for the symmetrisation in the two indices and removal of the trace: $S_{\mu\nu}\mathcal{O}_{\mu\nu} = \frac{1}{2}\mathcal{O}_{\mu\nu} + \frac{1}{2}\mathcal{O}_{\nu\mu} - \frac{1}{4} g_{\mu\nu}\mathcal{O}_{\xi\xi}$. The distribution amplitudes $\phi_g^T(t)$ and $\phi_g^S(t)$ are

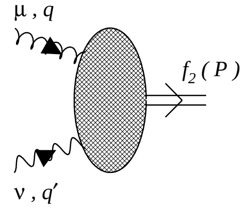

Figure 1: The kinematics of the process $\gamma^*(q) + \gamma(q') \to f_2(P)$

both symmetric to the interchange of $t \leftrightarrow -t$ and describe the momentum fraction distribution of the two gluons in the f_2-meson having the same and the opposite helicity, respectively. The asymptotic distributions at large scales are equal to

$$\phi_g^{T,\mathrm{as}}(t) = \phi_2^{S,\mathrm{as}}(t) = \frac{15}{16}(1-t^2)^2 \,. \tag{10}$$

The constants f_g^T and f_g^S are defined through the matrix element of the local two-gluon operator:

$$\begin{aligned}\langle f_2(P,\lambda)|G^a_{\alpha\beta}(0)G^a_{\mu\nu}(0)|0\rangle &= f_g^T\Big\{[(P_\alpha P_\mu - \frac{1}{2}m^2 g_{\alpha\mu})\, e^{(\lambda)}_{\beta\nu} - (\alpha\leftrightarrow\beta)] - (\mu\leftrightarrow\nu)\Big\} \\ &+ \frac{1}{2}f_g^S m^2\Big\{[g_{\alpha\mu}\, e^{(\lambda)}_{\beta\nu} - (\alpha\leftrightarrow\beta)] - (\mu\leftrightarrow\nu)\Big\}.\end{aligned} \tag{11}$$

The constant f_g^T is renormalized multiplicatively [7, 8, 9], while f_g^S is mixed with f_q [10]:

$$\begin{aligned} f_g^T(Q^2) &= f_g^T(\mu^2) L^{-1+6N_c/\beta_0}\,, \\ f_q(Q^2) &= \frac{n_f}{n_f+4C_F}\Big(f_g^S(\mu^2)+f_q(\mu^2)\Big) - L^{\frac{2}{3}(n_f+4C_F)/\beta_0}\Big(f_g^S(\mu^2) - \frac{4C_F}{n_f}f_q(\mu^2)\Big), \\ f_g^S(Q^2) &= \frac{4C_F}{n_f+4C_F}\Big(f_g^S(\mu^2)+f_q(\mu^2)\Big) + L^{\frac{2}{3}(n_f+4C_F)/\beta_0}\Big(f_g^S(\mu^2) - \frac{4C_F}{n_f}f_q(\mu^2)\Big), \end{aligned} \tag{12}$$

where $L = \alpha_s(Q^2)/\alpha_s(\mu^2)$, $C_F = \frac{N_c^2-1}{2N_c}$ and $\beta_0 = 11/3N_c - 2/3n_f$. Note that the sum $f_g^S + f_q$ is scale-independent. We are not aware of any estimates of the numerical values of f_g^T, f_g^S. The main goal of this letter is to point out that the constant f_g^T can be measured in hard exclusive processes.

3. In the rest of the work we consider one particular hard reaction: the transition form factor $\gamma^*(q) + \gamma(q') \to f_2(P)$ for one real $q'^2 = 0$ and one virtual $q^2 = -Q^2$ photon. The amplitude of this process is related to the matrix element

$$T_{\mu\nu} = i\int d^4x\, e^{-ix(q-q')/2}\langle f_2(P,\lambda)|T\{j_\mu(x/2)j_\nu(-x/2)\}|0\rangle, \tag{13}$$

where $j_\mu = e_u\bar{u}\gamma_\mu u + e_d\bar{u}\gamma_\mu u + \dots$ is the electromagnetic current. The kinematics is illustrated in Fig. 1. Neglecting contributions that vanish after the multiplication by photon polarizations, the amplitude $T_{\mu\nu}$ can be parametrized in terms of the three invariant form factors

$$
\begin{aligned}
T_{\mu\nu} = & -g^{\perp}_{\mu\nu} e_{\alpha\beta}(q-q')^{\alpha}(q-q')^{\beta}\frac{m^2}{(2qq')^2}\,T_0(Q^2)+ \\
& -g^{\perp}_{\nu\alpha} e_{\alpha\beta}(q-q')^{\beta}\Big(q-q'\frac{q^2}{(qq')}\Big)_{\mu}\frac{m^2}{(2qq')^2}\,T_1(Q^2) \\
& +\Big(g^{\perp}_{\mu\alpha}g^{\perp}_{\nu\beta}+g^{\perp}_{\mu\nu}\frac{(q-q')^2}{(2qq')^2}(q-q')^{\alpha}(q-q')^{\beta}\Big)e^{\alpha\beta}T_2(Q^2)\Big],
\end{aligned}
\tag{14}
$$

where we have introduced the metric tensor $g^{\perp}_{\mu\nu}$ that is transverse to the photon momenta

$$
g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{1}{(qq')}(q_\mu q'_\nu + q_\nu q'_\mu - \frac{q^2}{(qq')}q'_\mu q'_\nu)\,. \tag{15}
$$

The form factors T_0, T_1 and T_2 correspond to the three possible helicity amplitudes

$$
\begin{aligned}
T_0: &\quad \gamma^*(\pm 1)+\gamma(\pm 1)\to f_2(0)\,, \\
T_1: &\quad \gamma^*(0)+\gamma(\pm 1)\to f_2(\mp 1)\,, \\
T_2: &\quad \gamma^*(\pm 1)+\gamma(\mp 1)\to f_2(\pm 2)\,.
\end{aligned}
\tag{16}
$$

In the limit $Q^2 \to 0$ only T_0 and T_2 contribute and their values at $Q^2 = 0$ determine the two-photon decay width

$$
\Gamma_{\gamma\gamma} = \frac{(4\pi\alpha)^2}{60\pi m}\Big(|T_0(0)|^2 + 2|T_2(0)|^2\Big), \tag{17}
$$

where $\alpha = 1/137$ is the fine structure constant.

Experimentally it was found that the contribution of the helicity zero state to the two photon width $\Gamma_{\gamma\gamma}$ is very small (see, for example, [11, 12]). Using [11]

$$
\Gamma_{\gamma\gamma} = 3.15 \pm 0.04 \pm 0.39 \text{ keV}, \qquad \frac{\Gamma_0(f_2\to\gamma\gamma)}{\Gamma_2(f_2\to\gamma\gamma)} < 0.05 \ (90\%\,\text{C.L.}) \tag{18}
$$

we obtain

$$
T_2(0) = (212 \pm 20)\,\text{MeV}, \qquad T_0(0) < 67\,\text{MeV}. \tag{19}
$$

Next, we consider the region where the virtuality of one of the photons is large:

$$
-q^2 \equiv Q^2 \gg m^2,\ q'^2 = 0. \tag{20}
$$

In this kinematics, the form factors $\gamma^*(q) + \gamma(q') \to f_2(P)$ can be calculated by the light-cone expansion of the T-product of the electromagnetic currents in (13)

$$
iT\{j_\mu(x)j_\nu(-x)\} = \frac{1}{(4\pi)^2x^4}\sum e_q^2\Big\{\Big[\bar\psi(-x)\gamma_\nu \not{x}\gamma_\mu\psi(x) - \bar\psi(x)\gamma_\mu \not{x}\gamma_\nu\psi(-x)\Big] \tag{21}
$$

$$
-i\frac{\alpha_s}{4\pi}\int_{-1}^{1}du\int_{-1}^{1}dv\,(1+uv)S_{\mu\nu}G_{x\mu}(ux)G_{x\nu}(-vx)\Big\} + \ldots\,, \tag{22}
$$

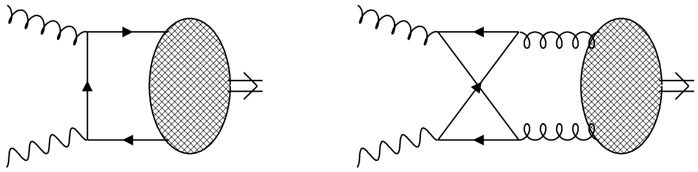

Figure 2: Leading-order contributions to $\gamma^*(q) + \gamma(q') \to f_2(P)$ in the large-Q^2 limit.

where we have shown the contributions that will be relevant for what follows, and $G_{x\mu} = x_\xi G_{\xi\mu}$, etc. The corresponding Feynman diagrams are shown in Fig. 2. Note that we take into account the contribution of the two-gluon operator with helicity $\lambda = 2$ but do not consider the gluon contribution with $\lambda = 0$ that enters at the same level as the $\mathcal{O}(\alpha_s)$ corrections to the quark-antiquark operator. In this paper we calculate the leading-twist contributions to the three form factors (helicity amplitudes) defined in (14) to the leading order in α_s. For the most interesting case of the gluon-dominated amplitude T_2 we also keep the leading higher-twist correction. A simple calculation gives:

$$\begin{aligned} T_0 &= \frac{5}{9} f_q \int_{-1}^{1} \frac{t dt}{1-t^2}\, \phi_q(t)\,, \\ T_1 &= \frac{5}{9} f_q \int_{-1}^{1} dt\, \ln(1-t^2)\, g_a^{WW}(t)\,, \\ T_2 &= 2\frac{\alpha_s}{\pi} \sum e_q^2\, f_g^T \int_{-1}^{1} \frac{dt}{1-t^2}\, \phi_g^T(t) + \\ &\quad + \frac{5}{9} \frac{f_q m^2}{Q^2} \int_{-1}^{1} dt\, \ln\left(\frac{1+t}{1-t}\right) g_v^{WW}(t)\,, \end{aligned} \tag{23}$$

where all distribution amplitudes and the coupling have to be taken at the scale $\mu^2 = Q^2$. Note that to ensure the gauge invariance the higher-twist distribution amplitudes must be taken, to our accuracy, in the Wandzura-Wilczek approximation.

It is worthwhile to mention that hard exclusive f_2-meson production can be viewed as a particular case of the more general hard exclusive two-pion production process considered in [13]. Factorisation theorems derived for the two-pion production [14] are valid for our case as well, and the results in (23), with the exception of the Wandzura-Wilzek contribution to T_2, can be extracted from the ones existing in the literature [13, 15, 16] using a correspondence between the two-pion soft matrix elements and the distribution amplitude of a tensor meson [17]. In particular, the hard parton subprocesses and hence the coefficient functions are the same in both cases.

We expect that distribution amplitudes of the f_2-meson are not far from their asymptotic form given in the text. On this assumption, which is certainly acceptable for an estimate, we obtain for large Q^2

$$T_2 \simeq \frac{5}{3} \frac{\alpha_s}{\pi} f_g^T + \frac{50}{27} f_q \frac{m^2}{Q^2}\,, \tag{24}$$

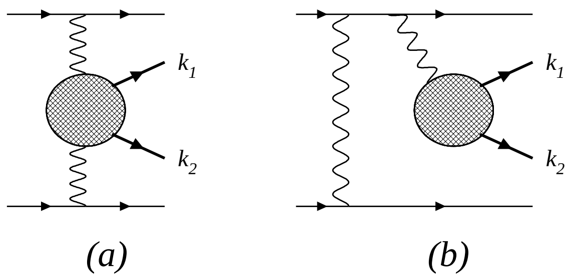

Figure 3: The two subprocesses contributing to the reaction $ee \to ee\pi^+\pi^-$: $\gamma^*\gamma$ scattering (a) and bremsstrahlung (b).

where we have taken into account u, d and s-quark contribution in the quark loop in the box diagram. For a realistic value $Q^2 \sim 10$ GeV^2 the gluon contribution is comparable to the subleading quark contribution if the constants f_g^T and f_q are of the same order. It is interesting to note that $T_2(0)$ is much larger than both $T_0(0)$ (19) and f_q (5). This may indicate that f_g^T is abnormally large.

Since f_2 decays in two pions with the branching ratio about 95%, one natural possibility to measure the form factors is via the hard exclusive two-pion production $ee \to ee\pi\pi$, see Fig. 3a, that received a lot of attention recently [13, 14, 15, 16, 18]. This reaction is in fact observed and constitites a background to the $\gamma^*\gamma \to \pi$ form factor measurements reported by CLEO [19]. Let k_1 and k_2 be the momenta of the two pions in the final state. The tensor form factor of interest $T_2(Q^2)$ is related to the two-pion helicity amplitude A_{+-} (we use notations of Ref. [18], see Eq. (75) there) in the region $(k_1+k_2)^2 \sim m^2$ as

$$A_{+-} = \varepsilon_+^\mu \varepsilon_-^{\prime\nu} (k_1-k_2)_\mu^\perp (k_1-k_2)_\nu^\perp \frac{g_{f_2\pi\pi}}{m} \frac{T_2(Q^2)}{m^2-(k_1+k_2)^2}, \tag{25}$$

where $g_{f_2\pi\pi}$ is the corresponding decay constant:

$$\langle \pi(k_1)\pi(k_2)|f_2(P,\lambda)\rangle = \frac{g_{f_2\pi\pi}}{m} e_{\alpha\beta}^{(\lambda)} (k_1-k_2)^\alpha (k_1-k_2)^\beta \tag{26}$$

and ε_+^μ, $\varepsilon_-^{\prime\nu}$ denote the transverse polarisation vectors of the virtual and the real photon, respectively. As was discussed in Ref. [18, 15] the amplitude A_{+-} can be separated using its nontrivial dependence on the azimuthal angle φ. Moreover, A_{+-} is symmetric to the interchange of the pion momenta and can be measured by the interference with the (much larger) contribution of the bremsstrahlung process (see Fig. 3b) that is antisymmetric to the interchange of the pion momenta: In the difference of cross sections $\sigma(k_1,k_2) - \sigma(k_2,k_1)$ only the interference term survives. The relevant expressions for the physical cross sections have been worked out in [18].

4. To summarize, in this letter we point out a possibility to determine gluon admixture in tensor mesons using hard exclusive reactions. We construct the basic theoretical formalism and suggest a particular experimental setup where the relevant form factor can be measured. For simplicity, in this work we did not consider radiative corrections. NLO corrections to the coefficient functions have been calculated in [15] and NNLO evolution of the corresponding distributions amplitudes can be found in [20, 21].

Acknowledgements

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